the elegance which we expect from him; but even he can scarcely find room for any facts which are not familiar to every well-informed person, or any conclusions which are not the commonplaces of every journalist.

While nothing but praise can be given to the authors for the performance of their tasks, the decision of the editors that all the modern historian requires to know of the science of the last century can be contained in two short chapters seems at first to challenge criticism. But, so far as pure science is concerned, we believe that they are right. Pure science is the most esoteric of all studies; the power of appreciating the value of the ideas contained in the most fundamental scientific theories appears to be totally uncorrelated with any other form of mental ability. It is just because the scientific instinct is such a rare and peculiar gift that it is so intensely valuable. Even if it were possible for the mass of mankind to know truly the meaning of science, it is very doubtful whether it would have any effect upon those actions which history studies. Science cannot define a worthy aim for action; at most it can show what aims can be attained, not what ought to be attained. It was, of course, thought very widely forty years ago that what was "natural" was good, but the fallacy is quite exploded to-day. Indeed, it is probable that Spencerian sociology, based on the confusion of the biological and ethical meanings of the word "fit," would have never received any serious attention, had not the doctrine that what has survived ought to have survived been so comfortable to those in authority. All that the doctrine of evolution can teach us in the matter of aims is that man is master of his destiny, that it is neither sufficient nor necessary to wait for the dispensations of a mysterious providence, but neither the science of the last or of the next century will decide the eternally disputed question of what that destiny ought to be.

However, though pure science cannot give us ends, applied science can give us means, and in respect of applied science "The Cambridge Modern History" appears to us defective. There is no connected account whatsoever of the great inventions or the progress of engineering during "The Latest Age." Mr. Whetham, concerned with pure science, naturally only mentions discoveries which have been the by-products of pure research. We have bare references to photography, dynamos, telegraphy, and two pages concerning medicine, surgery, and hygiene. But many of the inventions which have had the greatest economic or historical effects have no immediate connection with pure science. Can the historian of to-morrow analyse the events of to-day if he has never heard of such things as telephones, explosion engines, modern armaments, water-power, or electric lighting? The most trivial invention in appearance may revolutionise the world. Mr. Whetham rightly says that "the locomotive engine and the electric telegraph effected the great industrial and social revolution of the middle of the nineteenth century"; we suggest that it would not be ridiculous to claim for an invention so uninteresting technically and scientifically as the bicycle a comparable influence upon the end of it. Besides its economic effect in increasing men's radius of action, its social effect in furthering the independence of women must surely make it worthy of the attention of a student of the modern state; and yet the editors of "The Cambridge Modern History" have no official knowledge of it.

THE THEORY OF FUNCTIONS.

Introduction à la théorie des fonctions d'une variable. By J. Tannery. Deuxième édition; tome 2. Intégrales définies, Développements en Série, Langage géométrique, Fonctions de Variables imaginaires. Avec une Note de M. Hadamard. Pp. iv+480. (Paris: A. Hermann et fils, 1910.) Price 15 francs.

THE most interesting chapter in this volume is that which is entitled "Langage géométrique," especially from a pedagogic point of view. The main object of the treatise is to deduce everything from purely arithmetical assumptions; but as a practical teacher, Prof. Tannery was well aware of the value of diagrams as an aid to the imagination, or, as he puts it, for purposes of orientation. Consequently he has given a series of quasi-geometrical definitions, by means of which the ordinary formulæ and methods of analytical geometry are valid, and may be used practically for constructing diagrams to define boundaries of aggregates, &c. In the ordinary sense, of course, we thus get a locus corresponding to an equation $\phi(x,y)=0$; but in order to emphasise the fact that only arithmetical conditions are really imposed, the author replaces the term "locus" (*lieu*) by "bond" (*lien*), and practically confines this to the case where we may put $x=\phi(t)$, $y=\psi(t)$, ϕ, ψ being definite functions for a certain range of the real continuous variable t. The principal results of the chapter are the proof of the existence of simple contours in a plane, which separate it into two distinct continua (this is given after Mr. Ames), and the further conclusion that a domain which is $(m+1)$ times connex can be reduced to two simply connex domains by drawing $(m+1)$ simple curves.

Another notion that occurs in this chapter is that of the order of a point A with regard to a closed contour C. If a point P traverses C once in the positive direction, the variation of the amplitude of the vector AP is of the form $2k\pi$, where k is some integer or zero; and k is called the order of A with respect to C. This very important idea was generalised by Kronecker, and the present volume concludes with an important note by M. Hadamard (pp. 437–77) on some applications of Kronecker's index. The main point of this theory is that the property of a Jacobian, that it changes sign when two rows are interchanged, is brought into connection with topology (or *analysis situs*) in a very general sense. Every advance in the analytical treatment of this subject is noteworthy; because it is here that the contrast between geometrical intuition and analytical proof is so often a glaring one. For example, take Minding's surface, which is obtained by taking a strip of paper, giving it a half-twist, and then pasting the free ends together. It is easy to see that this surface has only

one boundary and only one face, and that this property remains if the surface is "deformed" in the usual sense of that word. But it would be very troublesome to prove these facts analytically, and an ordinary person finds it hard to think of such a proof as being anything more than a superfluous *tour de force*.

Before leaving this chapter, attention may be directed to the use which is made of cuts (*coupures*) in the plane of reference. For certain purposes, as shown for instance by Hermite and Heine, it is more simple to use a plane with cuts in it than to construct a Riemann surface; and the beginner in function-theory may be recommended to master this method before proceeding to Riemann's.

The remaining chapters may be more briefly summarised. There is one on definite integrals introducing the indispensable notions of integrals by excess and defect, and functions of limited variation; one on development in series; two on complex variables and their functions; and one on the differentiation and integration of such functions. Very properly, the author has confined himself to well-known and comparatively simple functions as examples, and he has given figures to illustrate the conformal transformations effected by $\zeta = e^z$, $\zeta = \sin^{-1} z$, and so on. As incidental examples, we may mention the Eulerian functions, and Weierstrass's factorial formula for a function the zeros of which are given.

Mathematical teachers will doubtless find this work of great value, because it helps to suggest what is really practicable in a course of lectures on this subject. The fact is that the theory of aggregates, and the classification of functions, have become such an essential part of all analysis that they cannot be ignored. A serious mathematical student must understand what is meant by such terms as closed and open aggregates, limiting points, uniform convergence, &c., and know something of the conditions for differentiation and integration. But if, with excess of zeal, the lecturer tries to bring out every point, and to be impeccably logical, he runs a serious risk of boring his audience. G. B. M.

PHYSIOLOGY OF REPRODUCTION.

The Physiology of Reproduction. By Dr. F. H. A. Marshall. Preface by Prof. E. A. Schäfer, F.R.S., and contributions by Dr. W. Cramer and Dr. J. Lochhead. Pp. xviii+706. (London: Longmans, Green, and Co., 1910.) Price 21*s*. net.

THE progress of science is very haphazard. For centuries the problems of breeding and heredity have engaged the attention of intelligent men, but it is only within the last five decades that any accurate scientific knowledge on the subject of breeding has been reached. Even now almost all our knowledge of this subject has been derived chiefly from a study of the results of the mating of two germ-cells. This one-sidedness is due to the processes which take place during reproduction being so little known.

This unequal advancement of the sciences is seen again in physiology. Here the physiology of digestion, of muscular contraction, of the nervous system, has been extensively studied, but that of reproduction has been largely neglected. Dr. Marshall has set himself to remedy this defect, and has collected all the available information on the subject. Reproduction incidentally concerns many different categories of people, and they have recorded their observations in numerous and varied periodicals and books. It must have been a task of considerable magnitude for the author to have collated all his sources of information; a glance at the references to quoted literature will convince us of this. The bibliography is in consequence not the least valuable part of the book.

But Dr. Marshall's volume is not merely a digested abstract of scattered papers taken from a great many journals; the author's own extensive work has given him an insight into his subject which enables him to impart information to his readers in a clear and lucid way. He has compiled a treatise which will remain a standard work for some time to come.

The work is to a great extent morphological, but, as everyone knows, it is absolutely necessary to have a sound knowledge of the structure of any organ before physiology is investigated. In studying such a process as the œstrus cycle, the exact morphological changes which take place in the uterus must first be known. This knowledge we owe to Heape, to whom we are glad to see the volume is dedicated, and it is the foundation of all our knowledge of the physiology of the œstrus cycle.

The author commences with a chapter on the breeding season of animals, examples being taken from invertebrates as well as vertebrates, but in subsequent chapters the higher mammalia are, with few exceptions, alone referred to. He then deals with the œstrus cycle and the changes which take place in both sexes both before and during reproduction. After this we have detailed all the evidence bearing on the changes in the maternal organs during pregnancy. The chapter on the biochemistry of the sexual organs seems only to show us how little we know, the information in it is so sparse and disconnected, and surely here is a valuable field for research. The last three chapters are on general questions of breeding, such problems as fertility and the determination of sex being dealt with.

Interspersed with the subject-matter we find the author's views on many theories as seen in the light of his own research. Thus, in the chapter on fertilisation, he gives a criticism of Mendelism. Close study of the actual physiology of reproduction does not lead the author to believe in the conception of unit characters, which the Mendelians have put forward. He speaks of the idea of attempting to locate latent characters of organisms in different parts of the germ-cell, as a survival from the times when all qualities, abstract or otherwise, were supposed to reside in different portions of the body. Whatever be the merits of this particular criticism, it seems that when further work has been done on the physiology of reproduction, a new mechanism may have to be supposed which will account for Mendelian facts, and fit in as well with the teachings of physiology.

Dr. Marshall has produced a masterly treatise